

**Line  
Integrals!  
(Scalar)**

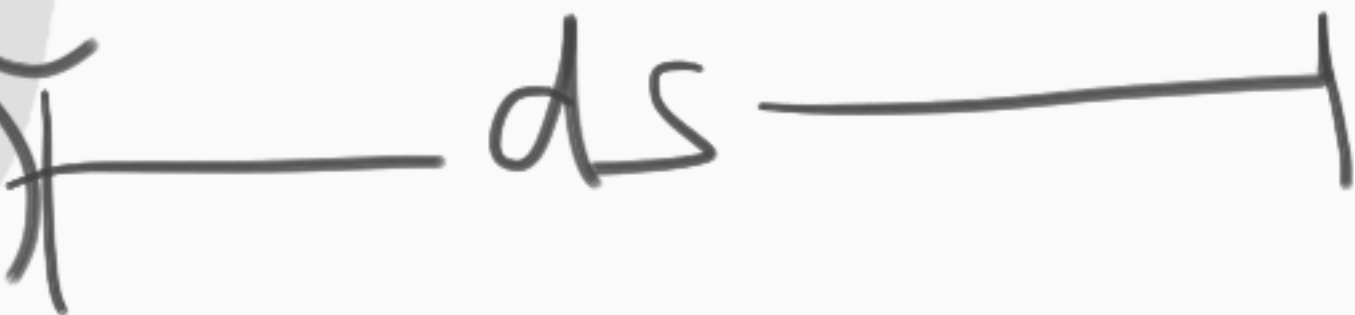
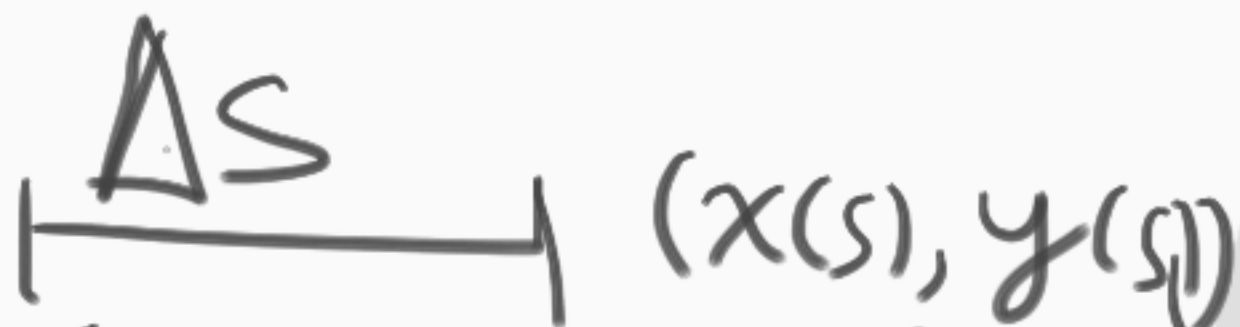
**Def**



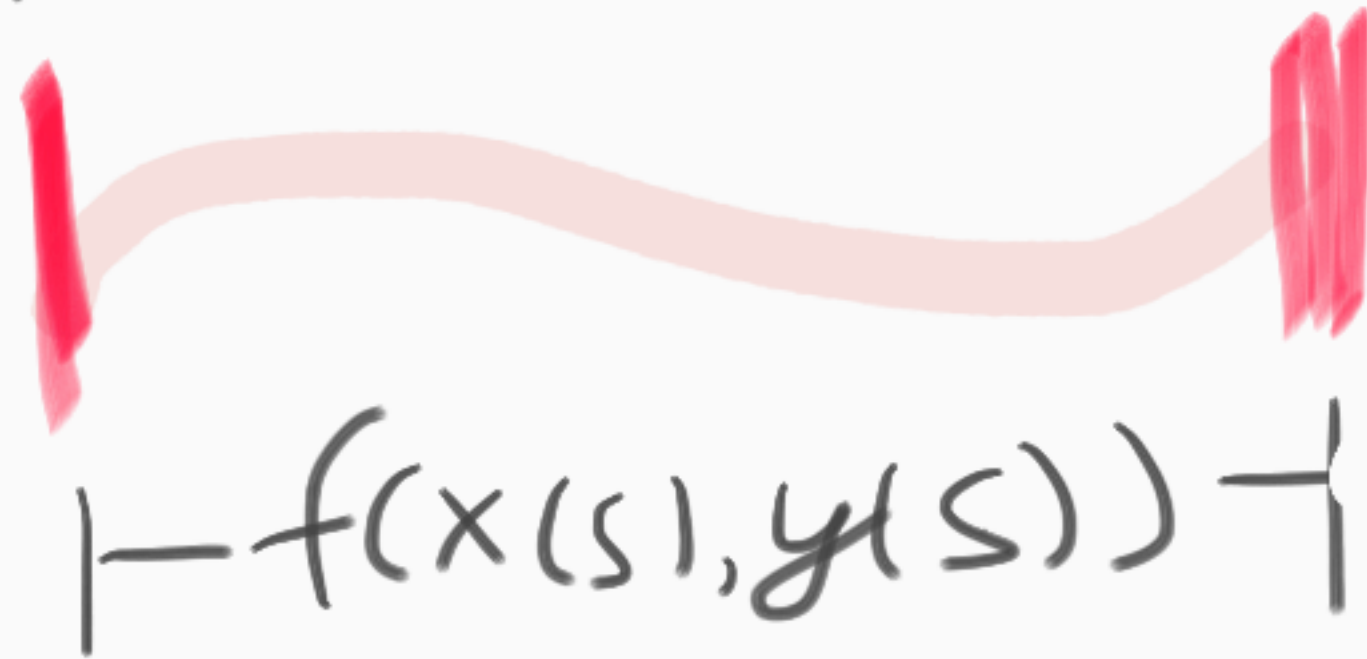
Physics  
Motivation

GOAL:  $\Sigma$  charge = ?

$$f(x, y) = f(x(s), y(s))$$



$$f(x(s), y(s)) ds$$



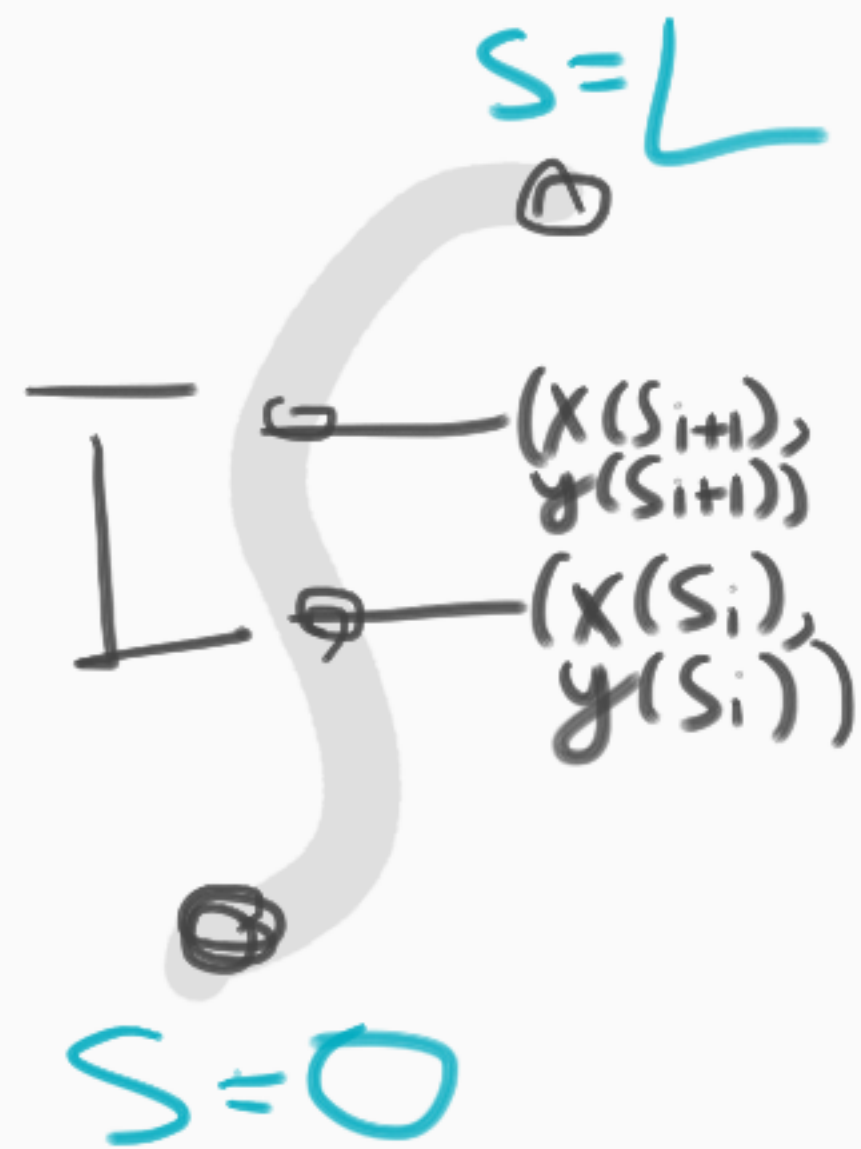
Physics  
Motivation

GOAL:  $\Sigma$  charge = ?

$$f(x(s), y(s)) \Delta s$$

$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x(s_i), y(s_i)) \Delta s$

$$\int_{s=0}^{s=L} f(x(s), y(s)) ds$$



## Notation

$$\int_{s=0}^{s=L} f(x(s), y(s)) \underline{\underline{ds}}$$

$$f(\underline{x(t)}, \underline{y(t)})$$

$$s = s(t)$$

$$ds = \underbrace{s'(t)} dt$$

$$ds = |r'(t)| dt$$

$$\int_{t=a}^{t=b} f(x(t), y(t)) |r'(t)| dt$$

**Example**

$$\vec{r}(t) = \langle \cos t, \sin t, t \rangle$$

$$0 \leq t \leq 2\pi$$

$$f(x, y, z) = x^2 + y^2 + z^2$$

$$\int f \, ds$$

$$ds = |\vec{r}'(t)| \, dt$$

$$\vec{r}'(t) = \langle -\sin t, \cos t, 1 \rangle$$

## Example

$$|r'(t)| = \sqrt{(-\sin t)^2 + (\cos t)^2 + 1}$$

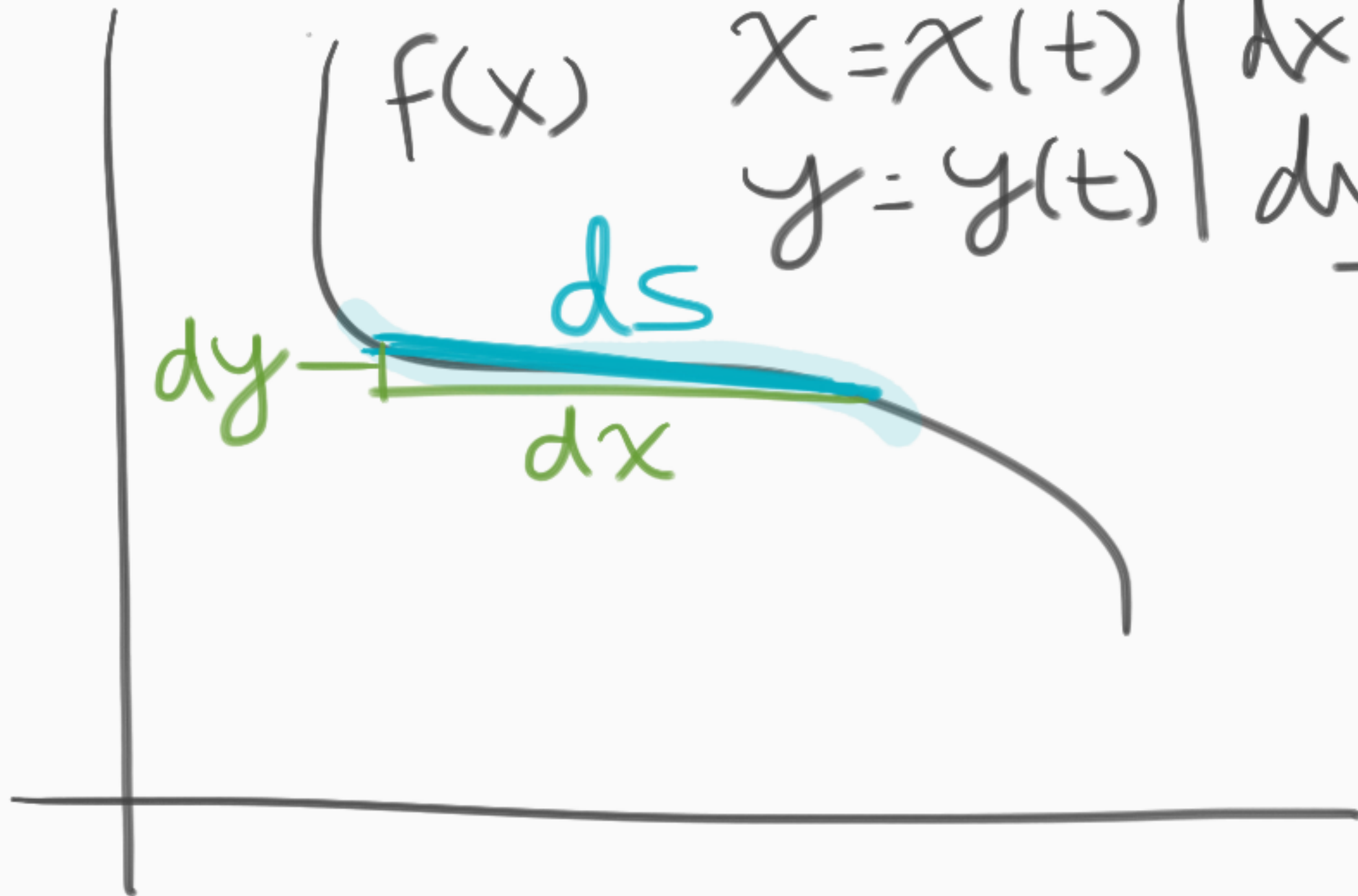
$$= \sqrt{2}$$

$$\int f ds \quad \left| \begin{array}{l} ds = |r'(t)| dt \\ r'(t) = \langle -\sin t, \cos t, 1 \rangle \end{array} \right.$$

# Arc Length

$$ds = \sqrt{(dx)^2 + (dy)^2}$$

$$x = x(t) \quad | \quad dx = x'(t) dt$$
$$y = y(t) \quad | \quad dy = y'(t) dt$$





# Arc Length

$$ds = \sqrt{(dx)^2 + (dy)^2}$$

$$x = x(t) \quad | \quad dx = x'(t) dt$$

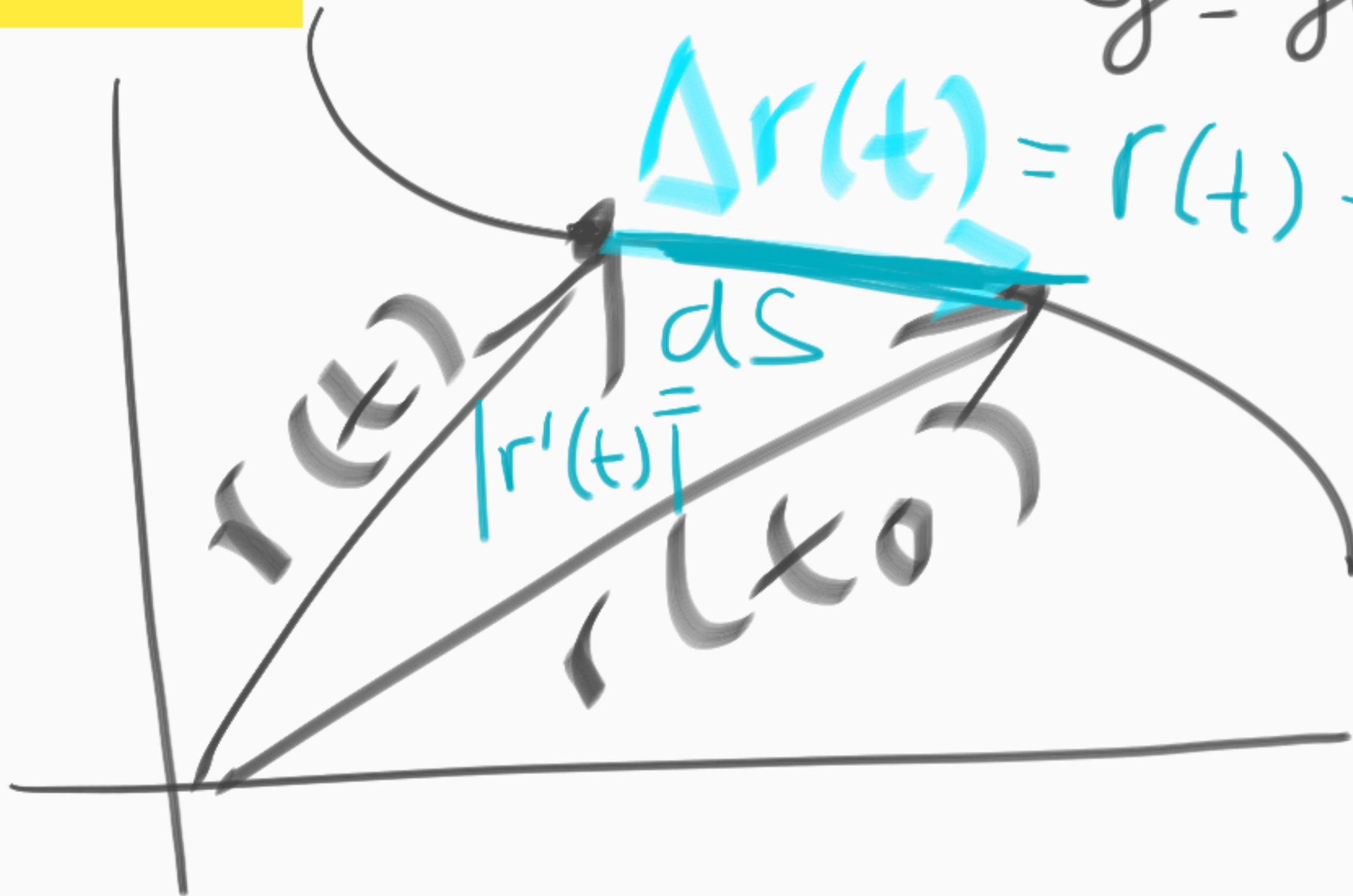
$$y = y(t) \quad | \quad dy = y'(t) dt$$

$$\Delta r(t) = r(t) - r(t_0)$$

$$\text{as } r(t) \rightarrow r(t_0)$$

$$\Delta r(t) \rightarrow r'(t)$$

$$|r'(t)| = ds$$



Geometric  
Motivation

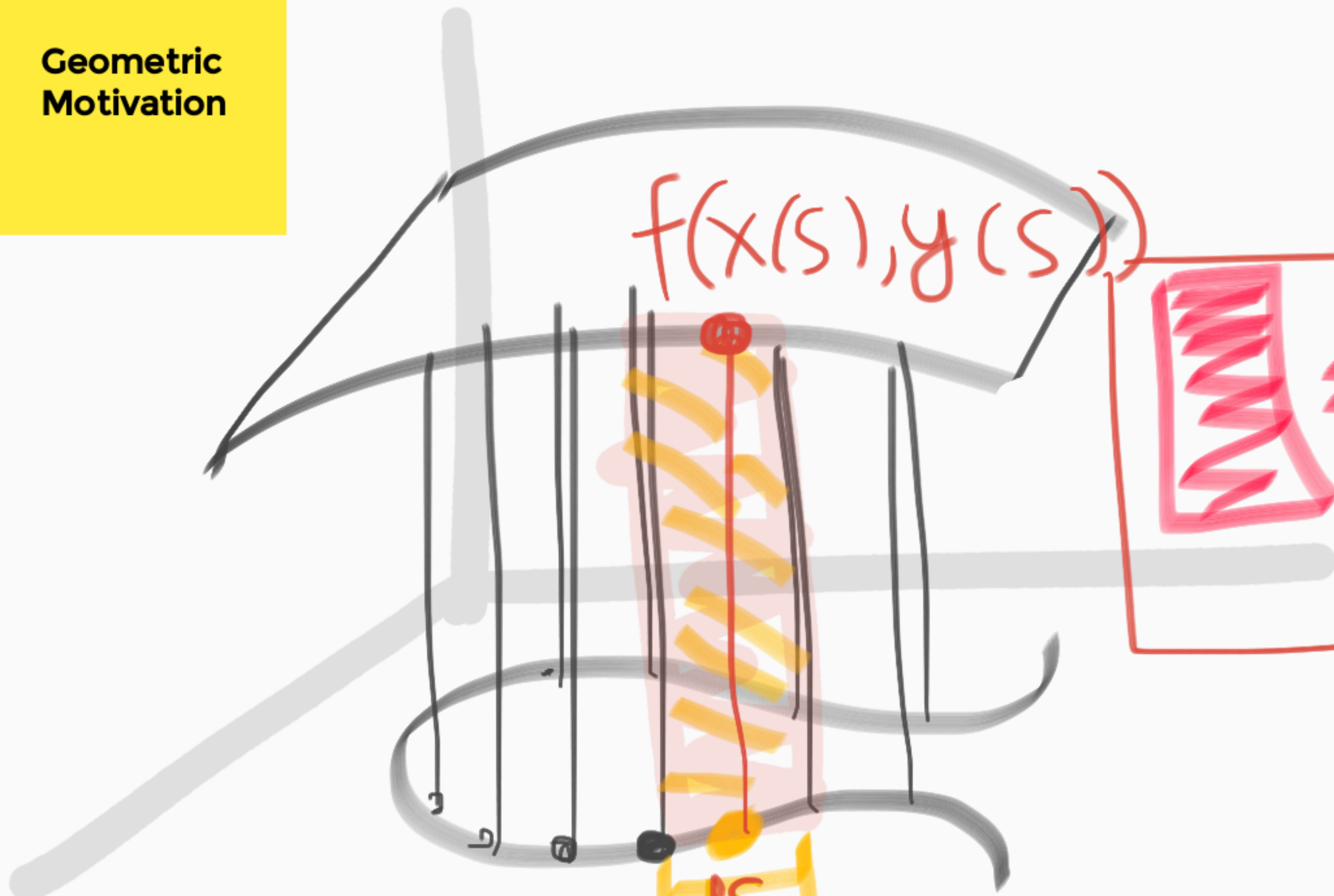
$$f(x(s), y(s))$$



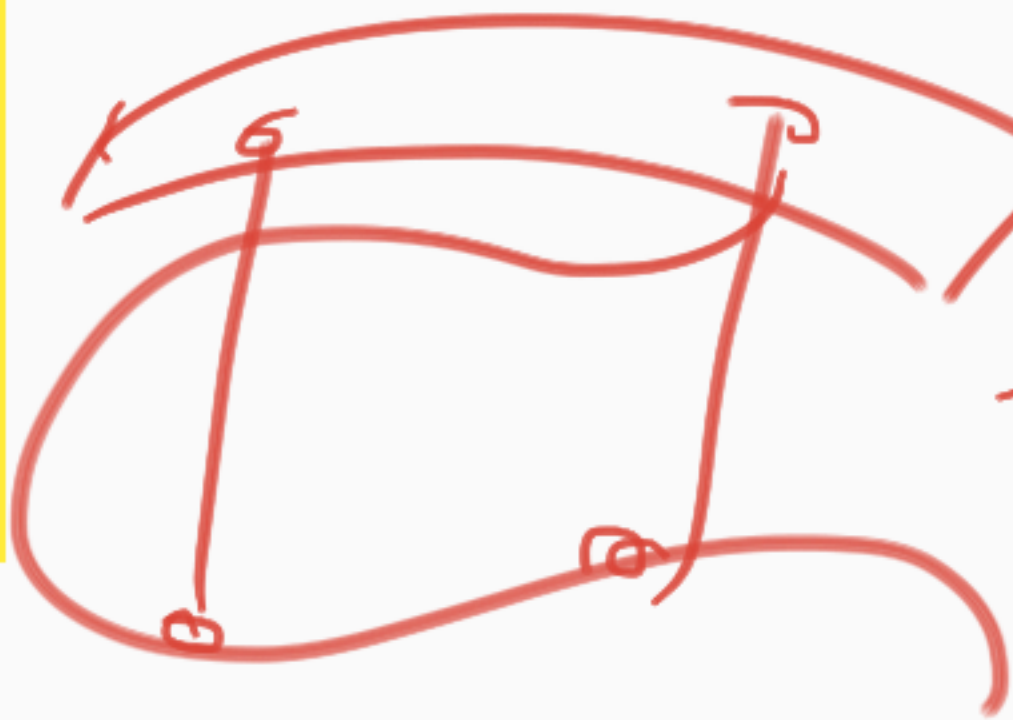
$$= ds$$

$$f(x(s), y(s))$$

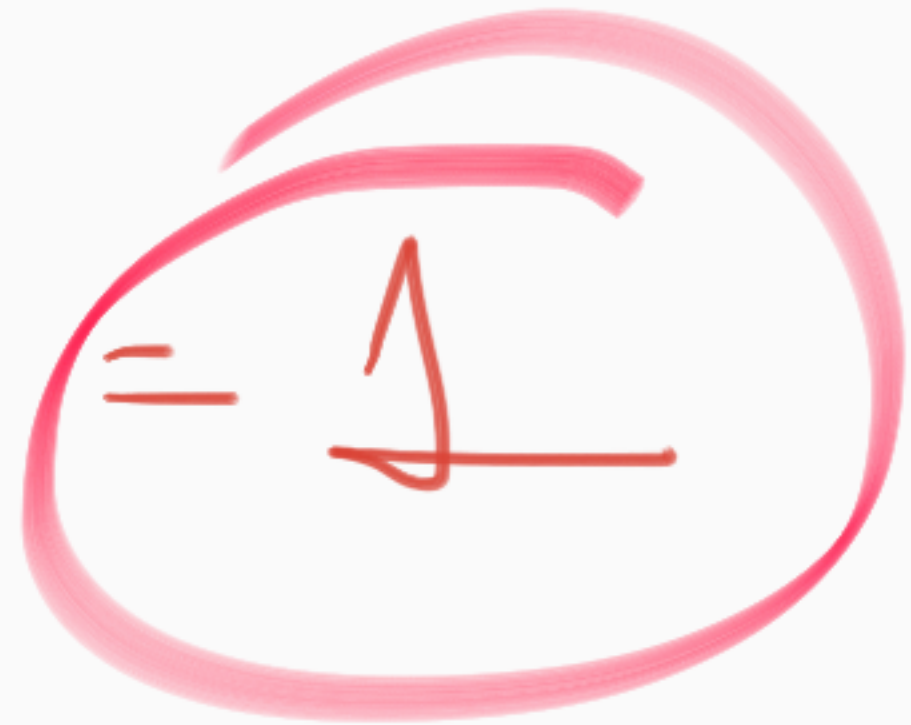
$$ds$$



**Remark  
on Length**



$$f(x, y)$$



$$\int f(x(s), y(s)) ds = ? \Rightarrow \int ds$$

1